COMMENTS

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Comment on "Inapplicability of the dynamic Clark model to the large eddy simulation of incompressible turbulent channel flows" [Phys. Fluids 15, L29 (2003)]

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Recently, Kobayashi and Shimomura[1] critized the use of the dynamic Clark model in large-eddy simulation of channel flow and provided the following reasons:

(1) An analysis revealed that the dynamic coefficient in the model does not satisfy the correct near-wall asymptotics.
(2) Negative diffusion was analytically identified in the viscous sublayer.
(3) Simulations with the model could not be completed due to instability.

However, this is not a representative result because the authors used a version of the gradient model which is only valid for isotropic filters, while their grid was strongly anisotropic and their test-filter in the dynamic procedure was also strongly anisotropic. If the anisotropic version of the gradient model is used, the three arguments listed above become invalid, as I will show in this Comment.

In the following, x_1, x_2, and x_3 correspond to the streamwise, normal, and spanwise direction, respectively. The basic filter defines a filter width $\bar{\Delta}_i$ for each direction. The filter can be isotropic,

$$\bar{\Delta}_1=\bar{\Delta}_2=\bar{\Delta}_3=\bar{\Delta}, \tag{1}$$

or anisotropic,

$$\bar{\Delta}_1\neq\bar{\Delta}_2\neq\bar{\Delta}_3, \tag{2}$$

and then usually $\bar{\Delta}=(\bar{\Delta}_1\bar{\Delta}_2\bar{\Delta}_3)^{1/3}$. The basic filter width often equals the grid-spacing, $\bar{\Delta}_i=h_i$ and the test-filter width in the dynamic procedure usually equals $\hat{\Delta}_i=2\bar{\Delta}_i$. The filter width of the test-filtered field, $\hat{\bar{\Delta}}_i$, is then defined by $\hat{\bar{\Delta}}_i^2=\bar{\Delta}_i^2+\hat{\Delta}_i^2$ for tophat filters.[2,3] In large-eddy simulation of channel flow the connection between grid and filter leads to anisotropic filters, due to the anisotropy and nonuniformity of the grid.

The dynamic Clark model, proposed by Vreman *et al.*,[2,3] results from substituting the model proposed by Clark *et al.*[4] in the Germano identity.[5] The Clark model is the sum of the gradient model and the Smagorinsky eddy-viscosity model. The gradient model (or nonlinear/tensor-diffusivity model) is obtained when Taylor expansions of the filtered velocity are used,[6]

$$\frac{1}{12}\bar{\Delta}^2\left(\frac{\partial\bar{u}_i}{\partial x_1}\frac{\partial\bar{u}_j}{\partial x_1}+\frac{\partial\bar{u}_i}{\partial x_2}\frac{\partial\bar{u}_j}{\partial x_2}+\frac{\partial\bar{u}_i}{\partial x_3}\frac{\partial\bar{u}_j}{\partial x_3}\right). \tag{3}$$

Reference 2 analytically proves the instability of this model in the Burgers equation, but also reports that the addition of a dynamic eddy-viscosity stabilizes the model in the simulation of a mixing layer.

It is important that expression (3) is valid for isotropic filters only, which causes no problems for the applications in Refs. 2–4. For anisotropic filters the Taylor expansion does not lead to Eq. (3), but to the gradient model derived in Ref. 2,

$$\frac{1}{12}\left(\bar{\Delta}_1^2\frac{\partial\bar{u}_i}{\partial x_1}\frac{\partial\bar{u}_j}{\partial x_1}+\bar{\Delta}_2^2\frac{\partial\bar{u}_i}{\partial x_2}\frac{\partial\bar{u}_j}{\partial x_2}+\bar{\Delta}_3^2\frac{\partial\bar{u}_i}{\partial x_3}\frac{\partial\bar{u}_j}{\partial x_3}\right). \tag{4}$$

The problems that Kobayashi and Shimomura[1] report are related to the use of the isotropic form (3) in combination with a nonuniform anistropic grid and an anisotropic test-filter.

Using the anisotropic form (4), all the conclusions and arguments in Ref. 1 change. First, the argument of improper scaling as given in Ref. 1 dissappears, because the second-order term in Eq. (24) of Ref. 1 falls out. That equation now becomes

$$L_{ij}+G_{ij}=O(\hat{\Delta}^4), \tag{5}$$

which holds for the anisotropic model in conjunction with two- or three-dimensional test-filtering. Here, $\hat{\bar{\Delta}}_2=\bar{\Delta}_2$ should be used for 2D test-filtering (=no test-filtering in the normal direction). The second-order term in Ref. 1 was entirely due to the fact that the isotropic form (3) was used together with a 2D test-filter (strongly anisotropic). If the term vanishes, the improper asymptotic scaling of the model coefficient as expressed by Eq. (30) in Ref. 1 is invalid.

Second, the interesting argument of negative diffusion caused by the gradient model changes. For the anisotropic form (4), the scaling of the dominant term in the gradient model in the viscous sublayer becomes (ignoring spatial variations in the filter width)

$$-\frac{\overline{\Delta}_2^2}{12}\mathrm{Re}_\tau \frac{\partial^2 \overline{u_i}}{\partial x_1 \partial x_2}. \tag{6}$$

This is similar to Eq. (34) in Ref. 1, with the notable difference that now Δ is replaced by $\overline{\Delta}_2$. The total diffusion coefficient in the laminar regime after a coordinate transformation by 45° can be calculated, like in Ref. 1. This results in a negative diffusion only if

$$\mathrm{Re}_\tau \overline{\Delta}_2 > 2\sqrt{6} \quad (\text{or } \overline{\Delta}_2^+ > 4.9), \tag{7}$$

where $\overline{\Delta}_2$ has been nondimensionalized with the channel half-width. In contrast to Eq. (37) in Ref. 1, Eq. (7) will not lead to problems in most large-eddy simulations of channel flow, because the normal grid-spacing (filter width) in the viscous sublayer is usually smaller than 4.9 viscous length-scales. Equation (7) is restricted to the viscous sublayer. Outside this layer, analysis would be more complicated. And there, the stabilization by the dynamic eddy-viscosity starts functioning.

Third, an example is presented of a large-eddy simulation of incompressible channel flow using the dynamic Clark model, but without any instability problems. The channel flow described in Ref. 1 is considered, corresponding to $\mathrm{Re}_\tau = 590$ and a domain $2\pi H \times 2H \times \pi H$. The grid is collocated and contains $33 \times 63 \times 33$ cells. It is nonuniform in the normal direction and stretched with a sinh-function.[7] There are three grid points between the wall and $y^+ = 10$. The dynamic Clark model[2] is used and its gradient component is given by Eq. (4). The tophat test-filter is applied in three directions and approximated with the trapezoidal rule using $\hat{\Delta}_i = 2\overline{\Delta}_i = 2h_i$, implying[3] $\hat{\overline{\Delta}}_i = \sqrt{5}\,\overline{\Delta}_i$.

The numerical method employs standard second-order central differences. The convective term is discretized in its well-known skew-symmetric form, which conserves kinetic energy. The implementation of this form at the walls is such that momentum is also conserved. The discrete velocities and pressure are defined at cell-centers, while the walls of the channel coincides with cell-faces. The discrete Poisson equation is consistent with the continuity equation and consequently involves the pressure values in (i,j,k), $(i\pm 2,j,k)$, $(i,j\pm 2,k)$, and $(i,j,k\pm 2)$. Odd–even decoupling (the "chessboard pattern") is not observed, because the pressure in odd and even cells can be coupled through the boundary conditions [R. W. C. P. Verstappen (private communication)]. In the present case, the wall-boundary condition of the pressure couples the odd to the even locations in the normal direction. The coupling in the periodic directions is caused by the odd number of cells.

The viscous terms are treated with the standard seven points discrete Laplacian operator. The subgrid-model is discretized according to formula (2.7) in Ref. 3. The time integration is performed using second-order Adams–Bashforth for the convective and Euler-forward for the viscous and subgrid-terms. The initial condition (provided by N. D. Sandham) is a power-law profile perturbed with a set of sinusoidal waves. The time step equals $0.001H/u_\tau$.

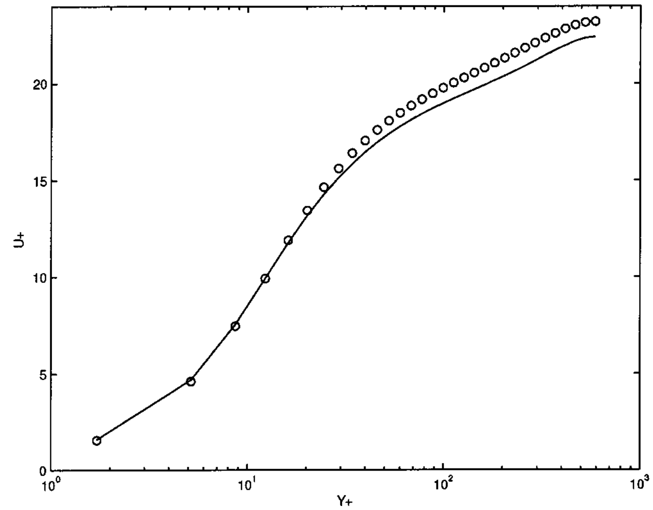

FIG. 1. Mean flow profiles resulting from large-eddy simulations of turbulent channel flow using the dynamic Clark model (Ref. 2) with a 3D (solid) and a 2D test-filter (circles).

Figure 1 shows a result of this simulation, which demonstrates that the dynamic Clark model can be applied to large-eddy simulation of turbulent channel flow. No instability problems were encountered, in contrast to Ref. 1, where the simulation could not be completed. Wall-damping was not used, in contrast to Ref. 8, which reported good results for a different version of the (anisotropic) dynamic Clark model. That version employed explicit filtering and a four times larger gradient component. Then Eq. (7) alters and becomes more critical: $\overline{\Delta}_2^+ > \sqrt{6}$.

Figure 1 adds a result for 2D test-filtering. That simulation was also stable, but compared to 3D test-filtering, the mean profile for 2D test-filtering is different (much too high). For 2D test-filtering, $\hat{\overline{\Delta}}_2 = \overline{\Delta}_2 = h_2$ was substituted into both the gradient and Smagorinsky component of the model on the test-filtered level. With respect to the dynamic assumption of similarity between filter levels, 3D test-filtering is more natural than 2D test-filtering, in case the basic filter is 3D.

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