Subgrid modeling in large-eddy simulation of complex flows

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1 Introduction

Two recent papers on large-eddy simulation of complex flows will be revisited and extended: a paper presenting a simply implementable subgrid model that is able to deal with transition and near-wall behavior [1], and a paper introducing the notion of adjoint filtering, which is required to ensure that the basic conservation properties of the Navier-Stokes equations are not violated after general nonuniform filtering [2]. Section 2 extends the model proposed in [1] to account for compressibility effects and presents advanced simulation results for turbulent supersonic mixing layers. Section 3 shows new adjoint modeling predictions for subgrid backscatter in turbulent channel flow.

2 Engineering subgrid model applied to supersonic flow

Ref. [1] presented an eddy-viscosity model which is essentially not more complicated than the Smagorinsky subgrid model. Unlike the Smagorinsky model, the proposed model is applicable to transitional flow and vanishes near walls. The model is expressed in first-order derivatives only and, in contrast to the well-known and successful dynamic model [3], it does not involve explicit filtering, ensemble averaging or clipping procedures. The foundation of the model relies on an algebraic classification of all three-dimensional flows. The construction of the model is such that the model and the theoretical subgrid dissipation vanish for the same classes of incompressible flows. In [1] the model was tested for a subsonic transitional mixing layer at high Reynolds number and for plane channel flow. In both cases the model outperformed the Smagorinsky model and was found to be as accurate as the computationally more demanding dynamic model. The range of flows tested for the new model is extended in this section.

LES with an eddy-viscosity closure solves the filtered Navier-Stokes equations, in which the unknown turbulent stress tensor, $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$, has been replaced by the model $-2\nu_e S_{ij}$, where $S_{ij} = \frac{1}{2}\partial_i \overline{u}_j + \frac{1}{2}\partial_j \overline{u}_i$. The following eddy-viscosity is considered in this section:

$$\nu_e = c\sqrt{Z/(\alpha_{ij}\alpha_{ij})}, \quad (1)$$

with model constant $c = 2.5C_S^2$, where C_S is the Smagorinsky constant, and

$$\alpha_{ij} = \partial_i \overline{u}_j = \frac{\partial \overline{u}_j}{\partial x_i}, \quad (2)$$

$$\beta_{ij} = \Delta_m^2 \alpha_{mi}\alpha_{mj}, \quad (3)$$

$$Z = \beta_{11}\beta_{22} - \beta_{12}^2 + \beta_{11}\beta_{33} - \beta_{13}^2 + \beta_{22}\beta_{33} - \beta_{23}^2 + (c_c \Delta \mathrm{div}\overline{u})^4/c^2, \quad (4)$$

where c_c is a constant and $\Delta^3 = \Delta_1\Delta_2\Delta_3$. The last term in Z is a compressibility correction. Originally the model was derived for incompressible flow and for incompressible flow the correction vanishes. Without compressibility correction, the model is zero for one-dimensional flow and, consequently, it does not dissipate one-dimensional shocks. This problem is cured by the compressible extension. For one-dimensional flow the compressible eddy-viscosity reduces to a Smagorinsky viscosity with Smagorinsky constant c_c. For the following supersonic application $c = 0.07$ and $c_c = 0.1$ was used.

Like the Smagorinsky model, the new model is well-suited for engineering applications, since it does not need more than the local filter width and the first-order derivatives of the velocity field. The symbol α represents the 3×3-matrix of derivatives of the filtered velocity $\overline{u}$. If $\alpha_{ij}\alpha_{ij}$ equals zero, ν_e is consistently defined as zero. The tensor β is proportional to the anisotropic gradient model and positive semidefinite, which implies $Z \geq 0$.

The WALE model [4], which was constructed to correctly reproduce near-wall behavior, is also simply to implement in complex flows. That model is based on the square of the velocity gradient matrix (α^2), which is fundamentally different from $\alpha^T\alpha$. Model (1) is based on $\alpha^T\alpha$, which is directly related to the formal definition of the turbulent stress tensor, through Taylor expansion.

Using model (1) we found an interesting result for the temporal mixing layer at convective Mach number $M_c = 2.0$, a flow which according to the knowledge of the author has never been simulated before. Initial Reynolds number (10^5) and other parameters are as in [1], where $M_c = 0.28$. Fig. 1 includes the momentum thickness for three supersonic cases: the dynamic model, the Smagorinsky model and model (1). For each case central differencing was used. The dynamic model quickly broke down due to numerical instability, whereas the mixing layer did not become turbulent at all when the Smagorinsky model was used, although C_S was low. However, the simulation with model (1) showed transition to turbulence, while it remained stable.

It is remarked that the Smagorinsky model was evaluated for $C_S = 0.1$ and $\Delta = h$, where h equalled the grid-spacing. Model (1) was evaluated for $c = 0.07$ and $\Delta = 2h$ instead of $\Delta = h$, for stability reasons in this demanding

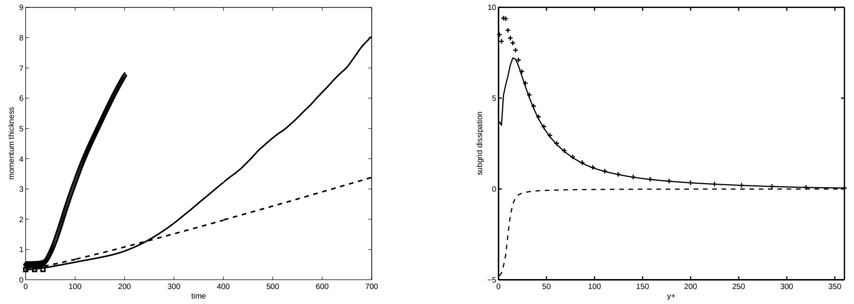

Fig. 1. LEFT: Evolution of momentum thickness in supersonic mixing layer at $M_c = 2.0$ for three subgrid models: Equation (1) (solid), Smagorinsky model (dashed), dynamic model (squares). The reference curve at $M_c = 0.28$ [1] is also shown (thick curve). RIGHT: subgrid dissipation of adjoint filtered model, Eq. (5), in LES of channel flow (solid), decomposed into forward scatter ('+') and backscatter (dashed).

supersonic high-Reynolds number flow. The dynamic model was evaluated for $\Delta = 2h$ as well, but this could not cure its stability problem.

The compressible growth-rate reduction simulated by model (1) is somewhat less than the reduction known from experiments. The turbulent growth-rate δ', determined as the slope of the momentum thickness and directly related to the integrated turbulent production [5], is about 0.043 for $M_c = 0.28$ [1] and 0.014 for $M_c = 2.0$. Thus, the reduction of the simulated growth-rate at $M_c = 2.0$, is about 70% of the low Mach number value. In an experiment at comparable Mach number ($M_c = 1.9$) the measured growth-rate was 78% reduced, compared to the incompressible measurement [6].

Finally, model (1) was tested in two-phase channel flow with high particle volume fraction and the results were found to be at least as accurate and as those documented in literature for the computationally more expensive dynamic model [7]. In conclusion, model (1) has been tested for a wide range of flows and in each case the results were quite good, compared to both Smagorinsky and dynamic model.

3 Nonuniform adjoint filters and backscatter

In practical applications, it is often desirable to use a filter width that depends on the spatial location. It is well-known that nonuniform filters do not commute with the spatial derivatives in the filtered equations. Due to the commutation problem, the filtered equations are in general not local conservation laws [2]. In particular the nonuniformly filtered velocity has lost the incompressible divergence-free property. With the notion of an adjoint filtering technique the violation of conservation properties can be restored in a global sense [2]. In addition such a filter can be applied to include backscatter into a subgrid model, while the globally dissipative behavior remains ensured, analytically. We illustrate this for the Smagorinsky model, but we could have used another eddy-viscosity equally well, for example the one presented in the previous section.

The adjoint-filtered Smagorinsky model reads [2]

$$\tau_{ij} = -F^a (2C_S^2 \Delta^2 |s| s_{ij}), \tag{5}$$

where $s_{ij} = FS_{ij}$, $|s|^2 = 2s_{ij}s_{ij}$ and $F = I - G$, $F^a = I - G^a$. G is an arbitrary explicit filter operator and G^a its adjoint. The definition of the adjoint operator partial integration proves that the total dissipation of the latter model is positive for arbitrary G. Nevertheless, the model is able to predict backscatter (locally negative regions of $-\tau_{ij}S_{ij}$). Model (5) is related to the variational multiscale approach [8] with two main differences. Firstly, the latter approach defines the large scales with use of a projection operator, which can always be written as a kernel filter [2]. Another difference with [8] is that in equation (5) the 'small-scale extraction' operator F is applied to the strain-rate and not to the velocity. Only in this way nonuniform filtering does not prohibit a tensorial form of the model. The tensorial form is necessary to use the standard expression for the definition of backscatter [9].

The turbulent channel LES configuration described in [1] was used to test model (5) for a three-points self-adjoint filter G (equation (52) in Ref. [2] was implemented for $\gamma = \frac{1}{2}$) with satisfactory results. Fig. 1 (right) clearly shows that the model predicts backscatter near the wall. The total amount of backscatter is about -13% of the entire subgrid dissipation.

References

1. Vreman AW (2004) An eddy-viscosity model for turbulent shear-flow: algebraic theory and applications. Phys Fluids 16, 3670
2. Vreman AW (2004) The adjoint filter operator in large-eddy simulation of turbulent flow. Phys Fluids 16, 2012
3. Germano M, Piomelli U, Moin P, Cabot WH (1991) A dynamic subgrid-scale eddy viscosity model. Phys Fluids A 3, 1760
4. Nicoud F, Ducros F (1999) Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow Turb Comb 62, 183
5. Vreman AW, Sandham ND, Luo KH (1996) Compressible mixing layer growth rate and turbulent characteristics. J Fluid Mech 320, 235
6. Papamoschou D and Roshko A (1988) The compressible turbulent shear-layer - an experimental study. J Fluid Mech 197, 453
7. Vreman AW, Geurts BJ, Deen NG, Kuipers JAM (2004) Large-eddy simulation of a particle-laden turbulent channel flow. In: Friedrich R, Geurts BJ, Metais O (eds) Direct and Large-Eddy Simulation V. Kluwer, Dordrecht, pp271-278
8. Hughes TJR, Mazzei L, Jansen KE (2000) Large eddy simulation and the variational multiscale method. Comp Visualiz Sci 3, 47
9. Piomelli U, Cabot WH, Moin P, Lee SS (1991) Subgrid-scale backscatter in transitional and turbulent flows. Phys. Fluids A 3, 1766